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An assessment of the accuracy of predicting the fundamental natural frequencies of buildings and the implications concerning the dynamic analysis of structures

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The accuracy of predicting the fundamental natural frequencies of buildings is examined. Predictions made using simple empirical formulae are shown to be as accurate as computer based methods, and a possible reason for the lack of accuracy in computer predictions is given. Several empirical formulae are examined, and the optimum formula for 163 buildings is derived. The significance of the non-linear characteristics and soil–structure interaction is considered, and the overall ramifications concerning the dynamic analysis of structures are discussed.

Introduction

The present trend of building more slender and lightweight structures has resulted in the problems associated with their dynamic behaviour becoming more apparent, and for some buildings it is gradually being realized that it is desirable to check the dynamic behaviour for the serviceability limit state. (For offshore structures and buildings in areas with a high seismic risk, their dynamic behaviour may also be important in the ultimate limit state.) The designer is therefore faced with calculating the response of a structure to various load conditions, and assessing the accuracy of his calculations.

2. This Paper examines the accuracy which can be expected in predicting the fundamental natural frequency of a building. The natural frequencies are often used as one of the basic input parameters for various methods of predicting the overall dynamic response of structures[1,2] and errors in input will undoubtedly result in errors in predicted structural behaviour.

Prediction of natural frequencies

Accuracy of computer predictions

3. In most buildings there are three fundamental modes of vibration which contribute significantly to the dynamic behaviour (one torsional mode and a pair of orthogonal translation modes). These modes may account for more than 90% of the overall motion caused by wind loading[3] and for most analyses of the dynamic behaviour of the whole structure, it is reasonable to disregard higher

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Table 1. Measured and predicted natural frequencies of ten buildings during the San Fernando earthquake, 9 February, 1971

Building	Number of storeys	Dimensions, m			Mode direction	Measured frequency, Hz			Frequency		Frequency by other models[1]	Types of modelling[1]	Reporters
		Height	Length	Width		Pre-earthquake	During earthquake	Post-earthquake	$f = 10/N$	$f = \frac{\sqrt{b}}{0{\cdot}091H}$			
Sheraton Universal Hotel	19	56	56	17·5	N	0·82	0·43	0·72	0·53	1·47	0·46	Girder, column and shear wall model	J. Blume & Associates[11]
					W	0·79	0·45	0·67		0·82	0·45		
Bank of California	12	48·5	49	18·5	N 11°E		0·45	0·59	0·83	1·59	0·55 1·17	Various frame and shear wall models	J. Blume & Associates[11]
					N 79°W		0·33	0·62		0·97	0·40 0·77		
Holiday Inn, Orion Avenue	7	20	49	19	N	2·08	0·62	1·47	1·43	2·39	1·07 1·85	Column, beam, slab model	J. Blume & Associates[11]
					W	1·89	0·81	1·39		3·82	1·16 1·56		
Holiday Inn, Marengo	7	20	49	19	N 38°W	1·89	1·00	1·56	1·43	2·39	1·16 1·56	Column, beam, slab model	J. Blume & Associates[11]
					S 52°W	2·04	0·83	1·58		3·82	1·07 1·85		
Bunker Hill Tower	32	103	38	27·5	N 53°W		0·30	0·39	0·31	0·66	0·28	Bare frame model	J. Blume & Associates[11]
					N 37°E		0·29	0·38		0·56	0·25		
KB Building (Venturia Gloria)	15	64·5	50·5	25	S 09°W		0·31	0·42	0·67	0·85	0·29	Lumped mass, column and shear wall model	Conrad Associates[11]
					S 81°E		0·36	0·44		1·21	0·30		
Muir Medical Centre	11	38	43·5	27	N	1·11	0·71	0·98	0·91	1·90	0·67	Lumped mass, column and shear wall model	Conrad Associates[11]
					E	0·97	0·62	0·88		1·50	0·62		
Kajima International Centre	15	58	30	20	N 36°E	0·76	0·34	0·48	0·67	1·04	0·30	Lumped mass, distributed stiffness model	Conrad Associates[11]
					N 54°W	0·53	0·36	0·47		0·85	0·31		
Certified Life Building	14	49	38	18·5	N 78°W	1·14	0·83	1·04	0·71	0·96	1·47	Lumped mass, distributed stiffness model	Conrad Associates[11]
					S 12°W	1·23	0·91	1·11		1·38	1·28		
Union Bank Square	39	151·5	60	30	N 52°W	0·35		0·24	0·26	0·40	0·20[1] 0·24[2]	1. Flexible joint model 2. Rigid joint model	Albert C. Martin & Associates[11]
					S 38°W	0·25		0·27		0·56	0·18[1] 0·21[2]		

Table 2. Measured and predicted natural frequencies of seven tall buildings

Building	Number of storeys	Dimensions, m			Mode direction	Measured frequency, Hz	Predicted frequency		Frequency by other models[1]	Types of modelling[1]	Reporters
		Height	Length	Width			$f = 10/N$	$f = \frac{\sqrt{b}}{0{\cdot}091H}$			
Health and Welfare Building, Ottawa	19	71·5	42·5	27		0·78	0·53	0·8	0·72–2·03	Frame action model, frame and core action	Crawford and Ward[16]
						1·01		1·00	0·71–2·13		
Canadian Imperial Bank of Commerce, Montreal	44	184	42·5	30·5	Perpendicular to long axis	0·22	0·22	0·33	0·31	Shear type structure with fixed columns	Ward and Crawford[17]
					Perpendicular to short axis	0·22		0·39	0·26		
CIL House, Montreal	34	131	51	34	Perpendicular to long axis	0·22	0·29	0·49	0·39	Shear type structure with fixed columns	Ward and Crawford[17]
					Perpendicular to short axis	0·25		0·60	0·33		
Post Office Building, Ottawa	10	45	81	22·5	Perpendicular to long axis	1·45	1·0	1·16	1·33	Shear type structure with fixed columns	Ward and Crawford[17]
					Perpendicular to short axis	1·69		2·20	1·11		
Canadian Department of Agriculture Building, Ottawa	11	40	94	22·5	Along short axis	0·89	0·91	1·30	0·51–1·73	Various frame and shear wall models	Ward[18]
					Along long axis	0·93		2·66	0·63–1·09		
37 storey building, Hawaii	37	99·8	23	23	N S	0·66	0·27	0·53	0·64	Lumped mass cantilever beam	Taoka *et al.*[19]
					E W	0·71		0·53	0·68		
27 storey building, Hawaii	27	73·4	62	18	Transverse	0·74	0·37	0·64	0·74	Equivalent frame model	Taoka *et al.*[19]
					Longitudinal	0·89		1·17	0·95		

Table 3. Correlation of measured and predicted lowest fundamental translation frequency for 17 rectangular plan buildings*

Predictor frequency proportional to	Correlation coefficient r	Best fit formula
$1/N$	0·9107	$12{\cdot}51/N$
$1/H$	0·9141	$41{\cdot}47/H$
$1/H^{0{\cdot}8}$	0·9172	$19{\cdot}45/H^{0{\cdot}8}$
$\sqrt{B}/H$	0·9011	$8{\cdot}87\sqrt{B}/H$
Computed values	0·8353	1·12 × prediction

* N = number of storeys; H = height of structure in metres; B = width of structure in metres.

Table 4. Effect of model complexity on computed frequencies[7]

Model complexity	First natural period, s	Frequency, Hz
1. Bared frame with K bracing	3·95	0·253
2. 1 + reduced girder depths	3·43	0·292
3. 2 + composite slab action	3·16	0·316
4. 3 + exterior panels	3·10	0·323
5. 4 + fire protection	3·06	0·326
Experimental	2·90	0·345

modes. There are several simple empirical formulae for predicting the frequency f of the fundamental translational mode; of these the simplest is $f = 10/N$, where N is the number of storeys. The formula $f = \sqrt{b}/0{\cdot}091H$, where b is the width of the building and H is the height in metres, is recommended by the Structural Engineers' Association of California.[4] It has long been realized that comparatively large errors are likely to occur using these simple formulae, but it has also been generally accepted that a satisfactory estimate of frequency can be obtained by using one of the standard, computer based methods.[5]

4. Tables 1 and 2 show details of 17 rectangular plan buildings together with the measured natural frequencies and frequencies predicted by using computer based methods. To show their relative accuracies, the predictions are compared with the actual measured values.* The correlation coefficients are given in Table 3, a higher value indicating better correlation. These show that for this sample the computed frequencies are less accurate than those obtained using the simple formulae. Tables 1 and 2 show that errors greater than ±50% are not abnormal.

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* In Table 1 the low amplitude pre-earthquake values were used in the correlation calculation; post-earthquake values were used only when the pre-earthquake values were not available. For the computer predictions, the average value was taken if a range of predictions was made, unless a specific preference had been stated.

Lack of accuracy in computer based predictions

5. Before the natural frequencies of a building can be computed, an idealized theoretical model of the building must be constructed. It is this theoretical idealization which is usually responsible for the major errors in the final result. The reason for the inadequacy of the idealization is that buildings are complicated structures and there is only a limited understanding of how they actually behave. Although the designer might have most of the available information about the structure, he will not be able to assess the effects of partition walls or cladding on the holistic behaviour of the structure, although these items provide additional stiffness.[6]

6. Some of the computed values in Tables 1 and 2 would have been obtained using simple theoretical models. However, using a more complex model will not necessarily result in a more accurate prediction. To obtain more accurate predictions, a better understanding of the overall behaviour of buildings is necessary, and this will be achieved only by comparing theoretical predictions with experimental measurements. Having obtained experimental data, it should be possible to alter the theoretical model to obtain a good correlation with the measured

Table 5, Correlation of measured and predicted lowest fundamental translation frequency for 163 rectangular plan buildings*

Predictor frequency proportional to	Correlation coefficient r	Best fit formula
$H^{-1{\cdot}5}$	0·8835	$220{\cdot}60H^{-1{\cdot}5}$
$H^{-1{\cdot}4}$	0·8851	$162{\cdot}60H^{-1{\cdot}4}$
$H^{-1{\cdot}3}$	0·8859	$119{\cdot}27H^{-1{\cdot}3}$
$H^{-1{\cdot}2}$	0·8860	$87{\cdot}10H^{-1{\cdot}2}$
$H^{-1{\cdot}1}$	0·8850	$63{\cdot}32H^{-1{\cdot}1}$
H^{-1}	0·8828	$45{\cdot}84H^{-1}$
$H^{-0{\cdot}9}$	0·8793	$33{\cdot}05H^{-0{\cdot}9}$
$H^{-0{\cdot}8}$	0·8743	$23{\cdot}72H^{-0{\cdot}8}$
$H^{-0{\cdot}7}$	0·8676	$16{\cdot}95H^{-0{\cdot}7}$
$H^{-0{\cdot}6}$	0·8591	$12{\cdot}05H^{-0{\cdot}6}$
BH^{-1}	0·5530	$2{\cdot}17BH^{-1}$
$B^{0{\cdot}5}H^{-1}$	0·7565	$10{\cdot}33B^{0{\cdot}5}H^{-1}$
$B^{0{\cdot}3}H^{-1}$	0·8217	$18{\cdot}90B^{0{\cdot}3}H^{-1}$
$B^{0{\cdot}1}H^{-1}$	0·8680	$34{\cdot}21B^{0{\cdot}1}H^{-1}$
$D^{0{\cdot}3}H^{-1}$	0·8846	$15{\cdot}35D^{0{\cdot}3}H^{-1}$
$D^{0{\cdot}2}H^{-1}$	0·8874	$22{\cdot}17D^{0{\cdot}2}H^{-1}$
$D^{0{\cdot}1}H^{-1}$	0·8869	$31{\cdot}92D^{0{\cdot}1}H^{-1}$
$D^{0{\cdot}2}H^{-1{\cdot}2}$	0·8918	$42{\cdot}22D^{0{\cdot}2}H^{-1{\cdot}2}$

* H = height; D = length; B = width; all dimensions in metres.

values for any one building. This type of study has been completed for some buildings; Table 4 presents the results of a study on one building.[7] It is to be hoped that one or two specific types of theoretical model obtained in this way will be found to be consistently the best predictors for the majority of a large number of buildings tested.

7. It is important not to place too much emphasis on any one set of results, but it is of utmost importance that for each study, both the theoretical model used for the predictions and the measured structural characteristics are well documented. Until such data are generally available it is unlikely that the accuracy of predicting natural frequencies will improve.

The best simple predictors

8. From the results presented, it appears that the simple predictors are, at present, the most accurate. To discover which is the best predictor, details of 163 rectangular plan buildings were collected. Different predictors were used to estimate the lowest fundamental translational frequency for each building and the results were correlated with the actual measured frequency. The results are given in Table 5. The predictor including the building width is worse than the simple height-dependent predictor, and therefore the predictor $f = \sqrt{b}/0{\cdot}091H$ is not the optimum for this set of data. For the sake of simplicity the predictor $f = 46/H$ is recommended (correlation coefficient $r = 0{\cdot}8828$), although a small improvement might be obtained using a more complex formula. Fig. 1 shows the results.

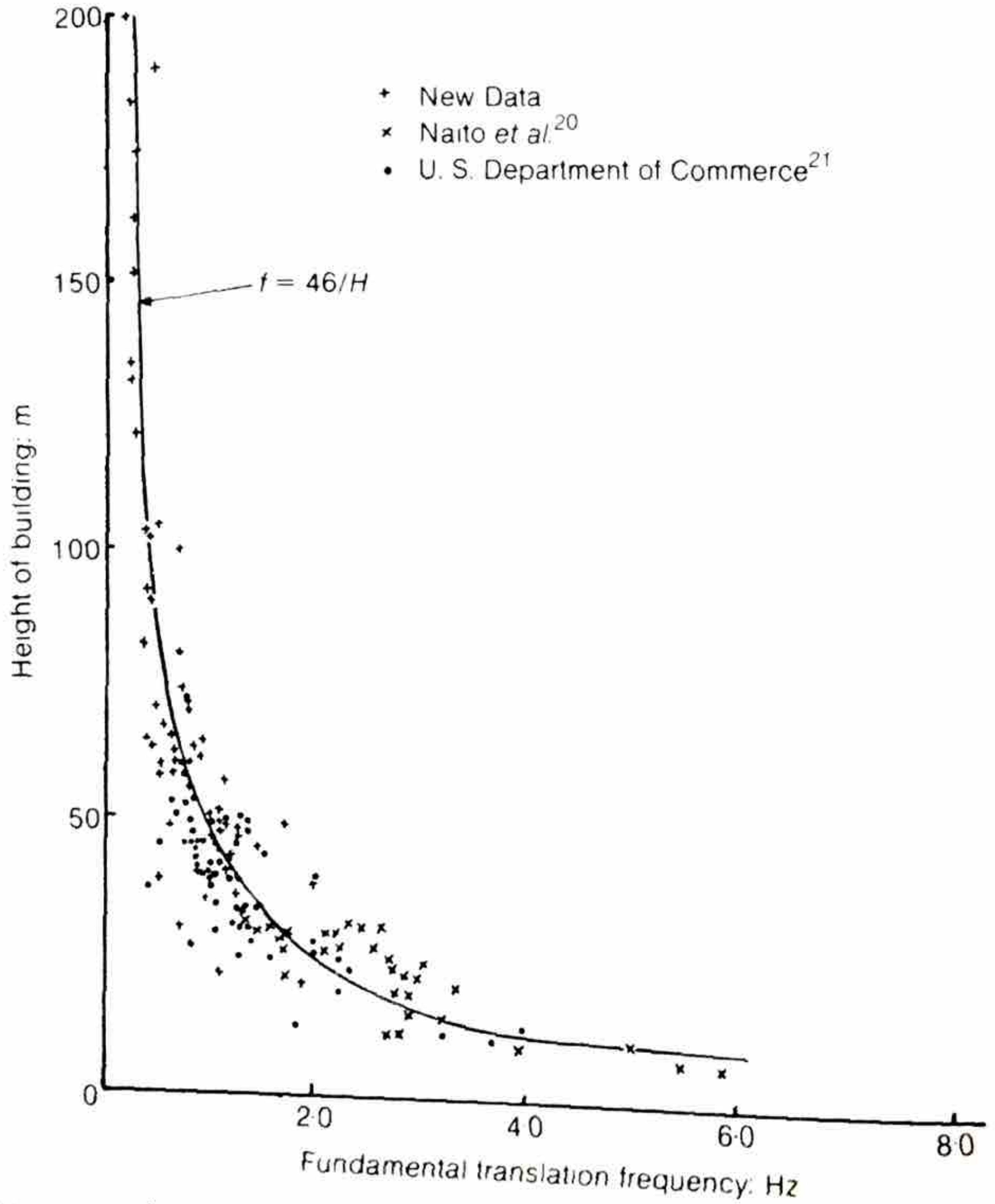


Fig. 1. Frequency plotted against height for 163 rectangular plan buildings

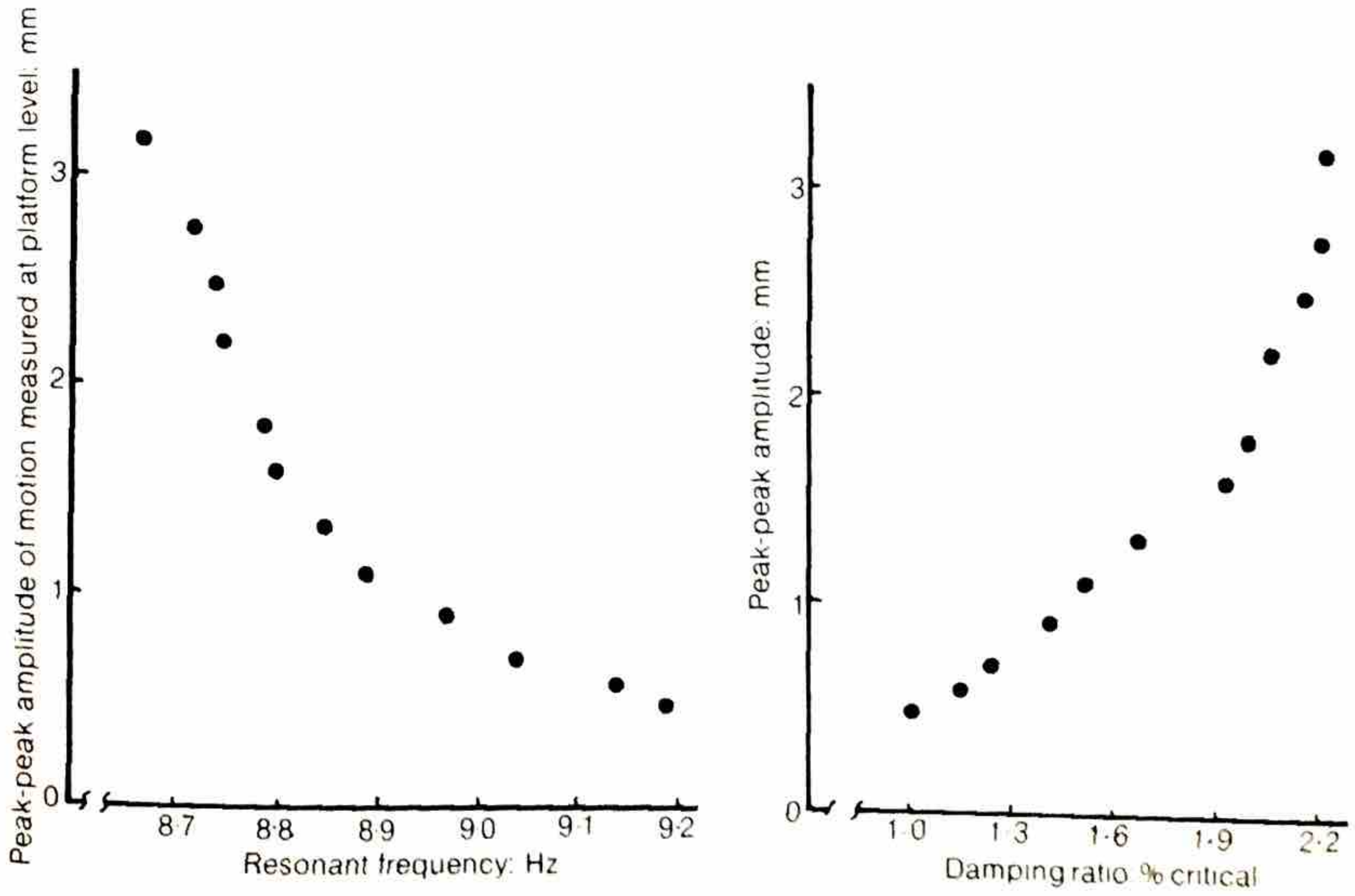
9. Similarly the frequency of the orthogonal fundamental translational mode can be estimated by $f = 58/H$ ($r = 0{\cdot}838$) and the fundamental torsional mode frequency can be estimated by $f = 72/H$ ($r = 0{\cdot}657$ for a sample of 63 buildings). Large errors are likely to arise with the use of any of these predictors, especially the torsional mode predictor.

10. The fundamental torsional mode can have a significant effect on overall structural behaviour,[6] and can be the cause of local damage (cracking in cladding and panels) around the periphery of the top storeys in a building. Therefore it is important to consider the effects of torsional vibration in any dynamic analysis.

The real characteristics of natural frequencies

Earthquake response and non-linear behaviour

11. The impression that natural frequencies are invariant is often given in published literature. However, the measured frequencies in Table 1 changed at the onset and the end of the San Fernando earthquake. Full-scale tests involving induced large amplitude motion of buildings[8,9] have also shown this non-linear behaviour, and recent tests have shown that the fundamental natural frequencies are actually dependent on the amplitude of motion, and exhibit an amplitude softening behaviour. Some results (Fig. 2) from a model scale experiment[10] illustrate this trend. As well as the amplitude dependent characteristics of the natural frequencies there is an amplitude dependence of the damping ratios. To



(a)

(b)

Fig. 2. Amplitude plotted against (a) frequency and (b) damping[10]

show that these effects should not be neglected, consider the behaviour of the Sheraton Universal Hotel during the San Fernando earthquake in 1971.[11] The fundamental frequency changed from a pre-earthquake value of 0·79 Hz to 0·45 Hz during the earthquake, and the damping ratio during the earthquake appeared to be about 10% critical, whereas most buildings have a damping ratio between 0·5% and 2·5% for low amplitude motion.

12. The amplitude dependence does not explain all the changes in frequency in Table 1, because the pre-earthquake and post-earthquake values show a change in frequency for similar amplitudes of motion. These results indicate a loss of stiffness in the structure and can be attributed to damage (or plastic deformation) caused by the earthquake.

Dynamic soil–structure interaction

13. There is a commonly held idea that computer based methods will provide good predictions of the natural frequencies of a building, and when differences occur between measured and predicted values a scapegoat is often sought. Dynamic soil–structure interaction is often suggested as a possible explanation for this deviation. The term dynamic soil–structure interaction is normally used to describe the effect of local soil conditions on the dynamic response of structures. If there is sufficient movement in the soil to make the natural frequencies of the structure significantly different from those of a similar structure on a rigid foundation, then the mode shapes will have observable displacements at ground level (see Fig. 3). For any particular mode, if the shape is similar to the fixed condition (i.e. no significant movement at the base), it can be assumed that soil–structure interaction is of no importance in this mode. When a building vibrates there will aways be movement in the soil, and hence a corresponding energy dissipation, both from internal damping and from the radiation of Rayleigh waves. However, in many cases this will not be significant.

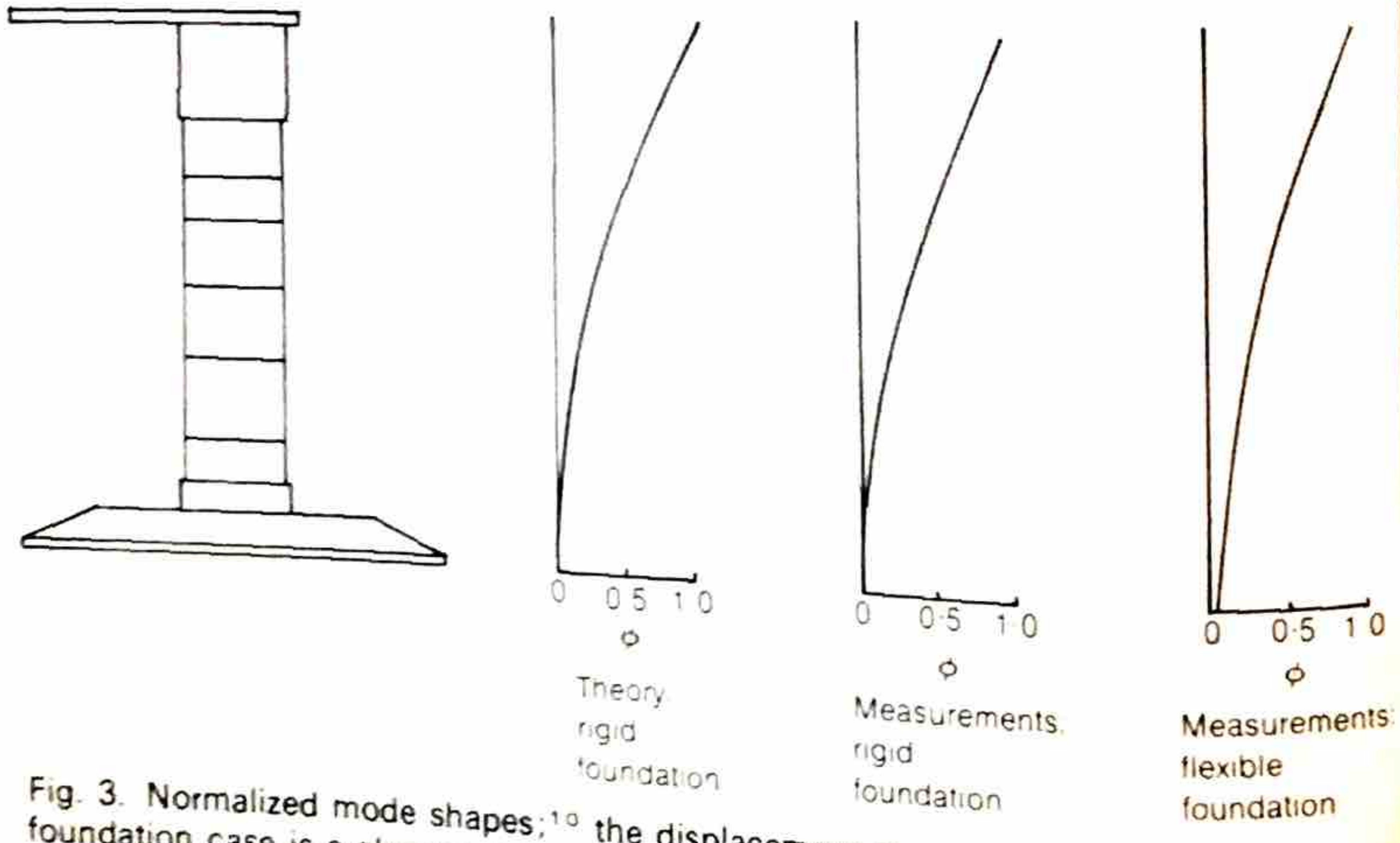


Fig. 3. Normalized mode shapes;[10] the displacement at ground level for the flexible foundation case is a characteristic of active soil–structure interaction

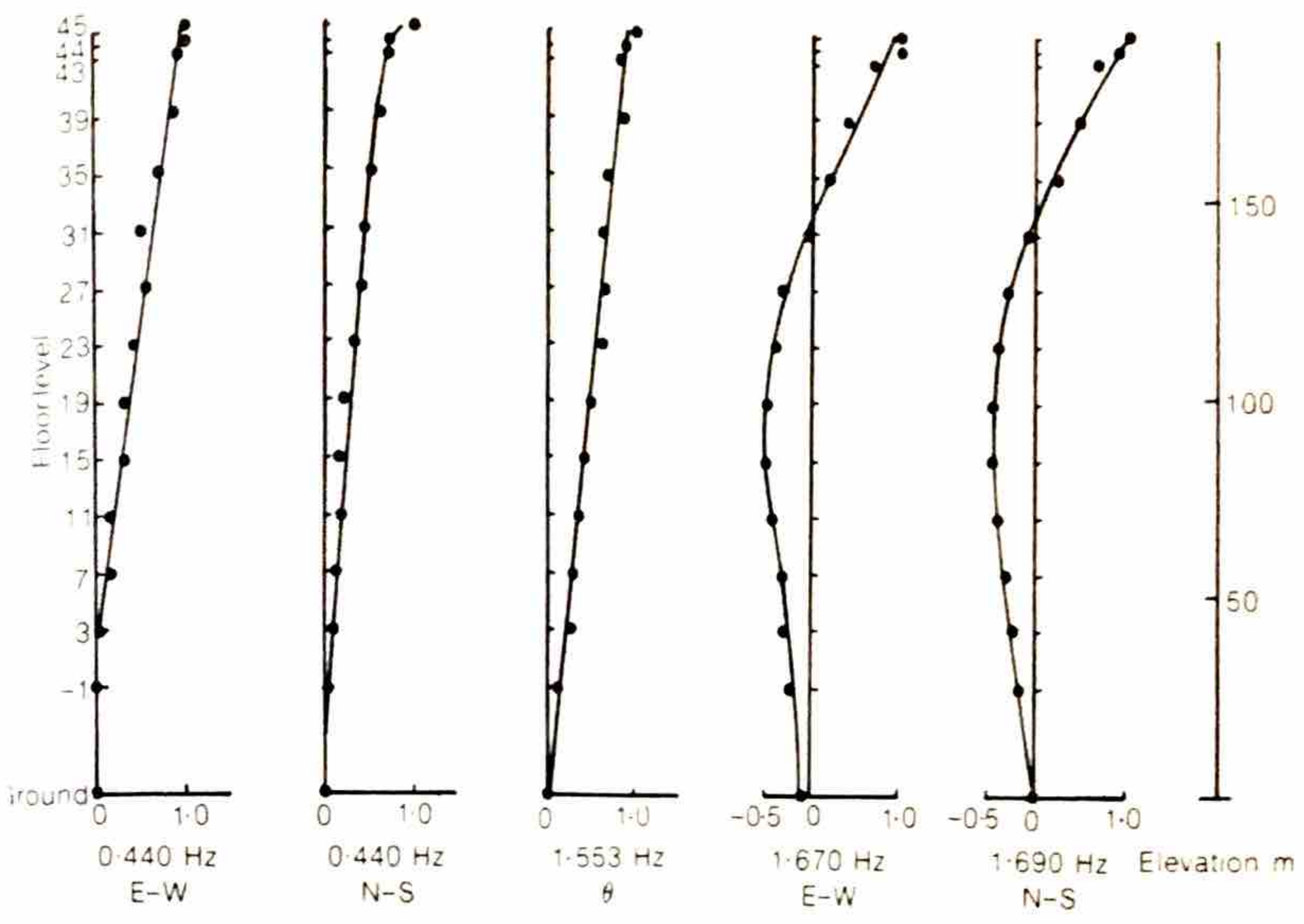


Fig. 4. Normalized mode shapes for fundamental translation, torsion and second translation modes of a 46 storey building

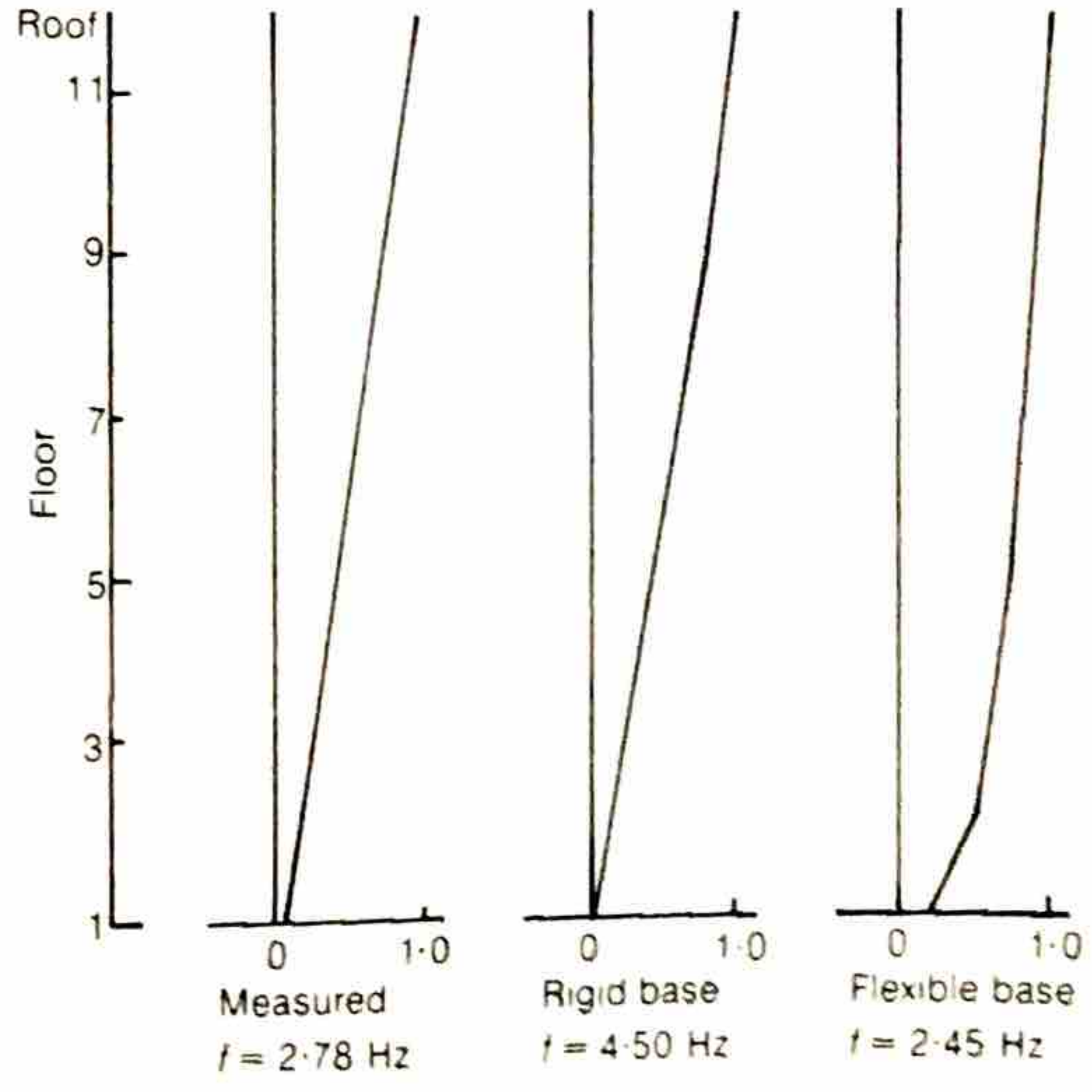


Fig. 5. Measured and predicted mode shapes for an 11 storey building[15]

14. By studying reports on dynamic tests on buildings in which the measured mode shapes are presented, it is possible to estimate whether soil–structure interaction was important. A sample of eleven buildings[3,7,12–14] shows no significant motion at the base in the fundamental modes, although in two cases there was observed motion at the base for the second modes. (Typical mode shapes for a building are given in Fig. 4.) However, these are all tall buildings, and soil–structure interaction may be more important in smaller buildings where the building/ground stiffness ratio is larger.

15. Soil–structure interaction plays an active role in the dynamic behaviour of the Oak Centre Towers in California, for which the measured and predicted frequencies and mode shapes are shown in Fig. 5. Although the flexible base solution (i.e. that including soil–structure interaction) has a predicted frequency within 12% of the measured frequency, the mode shapes are very different. This indicates that the assumed distribution of stiffness in the overall structural system was wrong, and that too much emphasis was placed on the foundation. It is stated by Stephen *et al.*[15] who tested the Oak Centre Towers that, 'The analysis of very rigid structures on flexible foundations must consider the soil–structure interaction phenomena, or the solution could be as much as 100% off.' Fig. 5 shows that the rigid base solution is much better at predicting the deflected shape, and hence the conclusion should simply read 'The analysis (prediction of natural frequencies) of structures can be as much as 100% in error.' Although soil–structure interaction was shown to be present, it was not the sole cause of errors in the computation.

Ramifications concerning the dynamic analysis of structures

16. As errors in predicting fundamental natural frequencies may be more than ±50%, even greater errors in the calculated overall response and stresses are to be expected. This indicates that any analysis involving the initial prediction of natural frequencies must be considered as approximate. The predictions of modal damping and the natural forces exerted on a structure are likely to be even more uncertain than the predicted frequencies, and so the possible errors in predicting the overall behaviour of a structure are considerable.

17. Even when the natural frequencies and damping values are known, the prediction of building response to wind loading can be over an order of magnitude in error.[3] Similar errors may be expected in the predicted response of structures to wave and earthquake loading, but for these cases the amplitude dependent characteristics of both natural frequencies and damping ratios may be significant.

18. The large uncertainty in predicting natural frequencies implies that dynamic soil–structure interaction can, at present, be ignored for tall buildings, because any difference in frequency between the rigid and flexible base predictions will be small in comparison with the likely error in the rigid base computations. (There is no evidence to suggest otherwise.)

19. Only the fundamental frequency modes have been discussed, but the predicted frequencies of higher frequency modes will suffer from similar or (more probably) greater errors. This means that, except for special cases where the mathematical model has been tuned to the experimental results, predictions involving many calculated modes must be regarded as unreliable.

20. The fact that as yet a reasonable model for an entire structure cannot be provided is not just a problem of dynamics, but equally one of statics, and until much more information has been gathered, designers should tread warily.

Conclusions

21. Errors of ±50% are not uncommon in the prediction of the fundamental natural frequencies of buildings. From the sample of 17 buildings considered it appears that the simple formulae are likely to be as accurate as computer based predictions.

22. Of the simple formulae available for predicting the fundamental translational mode frequency of a building, $f = 46/H$ seems to be the most reasonable for the sample of 163 rectangular plan buildings considered.

23. The amplitude dependent characteristics of both natural frequencies and damping can be significant both for offshore structures and for buildings in a zone of high seismic risk.

24. Dynamic soil–structure interaction can reasonably be ignored for the initial analysis of tall buildings, because most tall buildings so far tested show no indication of significant movement at ground level, and because the large uncertainties in predicting natural frequencies of rigid based structures are likely to overshadow the effect of soil–structure interaction.

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26. Table 4 and Fig. 5 are published by permission of the Director of the Earthquake Engineering Research Center, Berkeley, California, USA.

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